

Four dimensional black hole microstates: from D-branes to spacetime foam

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ABSTRACT: We propose that every supersymmetric four dimensional black hole of finite area can be split up into microstates made up of primitive half-BPS “atoms”. The mutual non-locality of the charges of these “atoms” binds the state together. In support of this proposal, we display a class of smooth, horizon-free, four dimensional supergravity solutions carrying the charges of black holes, with multiple centers each carrying the charge of a half-BPS state. At vanishing string coupling the solutions collapse to a bound system of intersecting D-branes. At weak coupling the system expands into the non-compact directions forming a topologically complex geometry. At strong coupling, a new dimension opens up, and the solutions form a “foam” of spheres threaded by flux in M-theory. We propose that this transverse growth of the underlying bound state of constituent branes is responsible for the emergence of black hole horizons for coarse-grained observables. As such, it suggests the link between the D-brane and “spacetime foam” approaches to black hole entropy.

KEYWORDS: D-branes, Black Holes in String Theory, Black Holes.

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1. Introduction

String theory has suggested two different pictures of the microstates underlying the entropy of black holes. The first, due originally to [1, 2], describes the underlying states as fluctuations of complicated bound states of string solitons. These analyses typically apply at very weak coupling, when there is no macroscopic horizon. A second picture (see the reviews [3, 4]) suggests that, at least in situations with sufficient supersymmetry, some of the underlying microstates can appear directly in gravity as a sort of “spacetime foam” [3–7], the details of which are invisible to almost all probes [8].¹ In this picture, the black hole with a horizon is simply the effective semiclassical description of the underlying “foam”.

¹For 1/2-BPS states in AdS₅ a similar picture has emerged in [9, 10]. Also see the related comments in [11, 12].

The present paper suggests how these two pictures are connected for four dimensional black holes — as the string coupling grows, D-brane bound states that form black hole microstates grow a transverse size, leading to a gravitational description as a topologically complex “spacetime foam”.

We provide evidence for this picture by constructing a large class of smooth, horizon-free, four dimensional supergravity solutions that have the charges of macroscopic black holes. Our construction proceeds by compactifying the five-dimensional (M-theory on T^6) solutions of [6, 5]. Typical geometries contain multiple centers, each carrying the charge of a 1/2-BPS state. The geometries are characterized by a region of complex topology containing a “foam” of spheres threaded by flux. This leads to a proposal: *Every supersymmetric four dimensional black hole can be split up into microstates made of 1/2-BPS “atoms”. The mutual nonlocality of the charges of these “atoms” binds the solution together.*

The geometric structures in our solutions scale as the string coupling is changed. As the string coupling g_s decreases, the region of complex topology shrinks until it is best interpreted in terms of wrapped D-branes whose separations are proportional to g_s . As g_s decreases further, the states can be described in the low-energy quiver gauge theory of a system of intersecting D-branes, following Denef [13]. Thus we arrive at a picture where quantum gravity microstates associated to a spacetime with fixed asymptotic quantized charges go through various transitions as the coupling is changed. Every microstate begins life at $g_s = 0$ as a ground state of an intersecting D-brane system. As we increase the coupling, the microstate makes a transition from a quiver gauge theory in the Higgs phase, to one in the Coulomb phase. As we further increase the coupling, a closed string picture becomes appropriate and, for states having a classical limit, we obtain the four dimensional solutions described in this paper. Still further increasing the coupling to large g_s opens up the eleventh dimension and we find a “spacetime foam” in M-theory of two-cycles threaded by flux [6, 5]. A similar flow from D-branes to “spacetime foam” has been noted in the topological string [14].

2. Review of five dimensional solutions

Here we review the candidate smooth, horizon-free microstates for black holes and black rings in five dimensions that were derived in [6, 5]. We will find candidate microstate solutions for four dimensional, finite area black holes by compactifying these geometries.

Basic setup: M-theory reduced to five dimensions on a 6-torus has 1/8-BPS solutions that only carry membrane charges. The general ansatz for such solutions was given in [15] following [16, 17]. The five dimensional non-compact space is written as time fibered over a pseudo-hyperkahler² base space (HK) which we require here to have a $U(1)$ symmetry. The metric takes the form:

$$ds_{11}^2 = -(Z_1 Z_2 Z_3)^{-2/3} (dt + k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds_{HK}^2 + ds_{T^6}^2, \quad (2.1)$$

²The metric is hyperkahler but we allow the signature of HK to flip. The overall metric remains non-singular and of constant signature because the Z_i will change sign simultaneously.

where

$$ds_{\text{HK}}^2 = H^{-1}\sigma^2 + H(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2). \quad (2.2)$$

Here k is a 1-form on the hyperkahler base. H is a harmonic function on the flat \mathbf{R}^3 parametrized by (r, θ, ϕ) with poles having integer residues. Likewise, σ is a one-form on \mathbf{R}^3 ($\sigma = d\tau + f_a dx^a$ where τ has period 4π) satisfying

$$\star_3 d\sigma = dH. \quad (2.3)$$

The Hodge dual \star_3 acts in the flat \mathbf{R}^3 only. The metric on the torus is

$$ds_{T^6}^2 = (Z_1 Z_2 Z_3)^{1/3} (Z_1^{-1}(dz_1^2 + dz_2^2) + Z_2^{-1}(dz_3^2 + dz_4^2) + Z_3^{-1}(dz_5^2 + dz_6^2)). \quad (2.4)$$

The Z_i 's are functions on the hyperkahler base space, and the associated 2-tori have volumes V_i . The gauge field takes the form:

$$C_{(3)} = -(dt + k) \wedge (Z_1^{-1} dz_1 \wedge dz_2 + Z_2^{-1} dz_3 \wedge dz_4 + Z_3^{-1} dz_5 \wedge dz_6) \\ + 2a^1 \wedge dz_1 \wedge dz_2 + 2a^2 \wedge dz_3 \wedge dz_4 + 2a^3 \wedge dz_5 \wedge dz_6, \quad (2.5)$$

where the a^i are one-forms on the base space. After reduction on T^6 the C-field leads to three separate U(1) bundles, with connections $\mathcal{A}^i = -(dt + k)Z_i^{-1} + 2a^i$, on the five-dimensional total space. Defining $G^i = da^i$, [15] show that the equations of motion reduce to the three conditions (here the Hodge operator refers only to the base space HK):

$$G^i = \star G^i, \quad d\star dZ_i = 2s_{ijk} G^j \wedge G^k, \quad dk + \star dk = 2G^i Z_i, \quad (2.6)$$

where we define the symmetric tensor $s^{ijk} = |\epsilon^{ijk}|$.

Solution: This system of equations can be completely solved in terms of seven harmonic functions in addition to H , defined using variables $r_p = |\vec{x} - \vec{x}_p|$, where \vec{x} is a coordinate in the \mathbf{R}^3 appearing in (2.2):

$$H = \sum_{p=1}^N \frac{n_p}{r_p}, \quad M_i = 1 + \sum_{p=1}^N \frac{Q_i^p}{4r_p}, \quad K = l_0 + \sum_{p=1}^N \frac{l_p}{r_p}, \quad h^i = \sum_{p=1}^N \frac{d_p^i}{4r_p}. \quad (2.7)$$

To achieve an asymptotically flat metric, we require the NUT charge

$$n_T = \sum_p n_p \quad (2.8)$$

to equal 1. (The standard radial coordinate for the asymptotic \mathbf{R}^4 is $R = 2\sqrt{r}$.) At each point p , the Q_i^p measure the membrane charges, n_p sets the Kaluza-Klein monopole charge, l_p measures the angular momentum associated to the U(1) isometry. The d_p^i make contributions to the total 5-brane dipole moment of the solution. In terms of these functions, the equations of motion can be solved, giving:

$$Z_i = M_i + 2s_{ijk} h^j h^k / H, \quad a^i = (h^i / H) \sigma + a_a^i dx^a, \quad d(a_{ia} dx^a) = -\star_3 dh^i, \\ k = k_0 \sigma + k_a dx^a, \quad k_0 = K + 8H^{-2} h^1 h^2 h^3 + H^{-1} M_i h^i \\ d(k_a dx^a) = H \star_3 dK - K \star_3 dH + h^i \star_3 dM_i - M_i \star_3 dh^i \quad (2.9)$$

Requiring the absence of pathologies in these solutions constrains the parameters in various ways.

Constraints: The requirement of smoothness (no curvature singularities) constrains the charges of the harmonic functions:

$$Q_i^p = -s_{ijk} \frac{d_p^j d_p^k}{2n_p}, \quad l_p = \frac{d_p^1 d_p^2 d_p^3}{16n_p^2}, \quad l_0 = -\sum_i \frac{\sum_p d_p^i}{4n_T} \equiv -\sum_i \frac{s^i}{4n_T}. \quad (2.10)$$

It is useful to define

$$\tilde{\lambda}_p^i = (d_p^i/n_p - s^i/n_T), \quad \Gamma_{pq} = \frac{\prod_i (n_p d_q^i - n_q d_p^i)}{n_p^2 n_q^2}, \quad (2.11)$$

in terms of which the closure condition for $d(k_a dx^a)$, which makes $k_a dx^a$ a globally well defined one-form, becomes

$$4 \sum_i n_p \tilde{\lambda}_p^i + \sum_{q=1}^N \frac{\Gamma_{pq}}{r_{pq}} = 0, \quad p = 1 \dots (N-1). \quad (2.12)$$

where $r_{pq} = |\vec{x}_p - \vec{x}_q|$. While (2.10) constrains the parameters of a smooth solution, (2.12) determines the relative separations of the poles in (2.7). For a fixed set of charges, there are $N-1$ nonlinear equations relating $3N-3$ variables (after we fix the center of mass), so if a solution exists for a given set of charges satisfying (2.10), the locations of the poles supporting the charges will typically be a function of $2N-2$ moduli. We can set three more free parameters (although these are not completely independent of the charges) by specifying the angular momentum J_L defined below and its orientation. Finally, the absence of closed timelike curves and horizons in the solution requires that

$$\left(Z_1 Z_2 Z_3 H - k_0^2 H^2 - g_{\mathbf{R}^3}^{ab} k_a k_b \right) > 0 \quad (2.13)$$

must be satisfied everywhere. This condition also guarantees smoothness on the $H=0$ surface³ and that the metric has constant signature. It is possible that (2.13) is implied by (2.12) via some kind of gradient flow argument, but this is not immediately evident.

Charges: The total membrane charges and angular momenta of the solution are:

$$Q_i = -\frac{1}{2} \sum_{p=1}^N n_p s_{ijk} \tilde{\lambda}_p^j \tilde{\lambda}_p^k, \quad J_R = \sum_{p=1}^N n_p \tilde{\lambda}_p^1 \tilde{\lambda}_p^2 \tilde{\lambda}_p^3, \quad (2.14)$$

$$J_L = 4 \left| \sum_{p=1}^N \sum_i n_p \tilde{\lambda}_p^i \vec{x}_p \right| = \frac{1}{2} \left| \sum_{pq} \Gamma_{pq} \frac{\vec{x}_p - \vec{x}_q}{|\vec{x}_p - \vec{x}_q|} \right|. \quad (2.15)$$

The solution also carries a net 5-brane dipole moment. Finally, the absence of Dirac strings in the C -field requires integral quantization conditions:

$$n_p, \frac{\pi \Gamma_{pq}}{4G_N^{(5)}} \in \mathbf{Z}, \quad d_p^i = m_p^i / e^i, \quad m_p^i \in \mathbf{Z}, \quad (2.16)$$

³At $H=0$ there is an ergosphere in the solution. However, unlike a rotating black hole there is no ergoregion.

with constants

$$e^i = \frac{V_i}{(2\pi)^2 \ell_P^3}, \quad \prod_i (e^i)^{-1} = \frac{4G_5}{\pi}. \quad (2.17)$$

Appropriate quantization of the membrane charge and angular momenta also imposes

$$m_p^i m_p^j / n_p \in \mathbf{Z}, \quad m_p^1 m_p^2 m_p^3 / n_p^2 \in \mathbf{Z}. \quad (2.18)$$

3. Reducing to four dimensions

Type II string theory compactified on a 6-torus has a spectrum of extremal supersymmetric black holes. The charge vector of these black holes transforms in the **56** representation of the $E_{7(7)}$ duality group. The entropy associated with the black hole horizons is a function of the quartic invariant of $E_{7(7)}$ constructed from the charges (see, e.g., [18, 19]). As a result, to have a finite entropy, these black holes must have at least four charges. Furthermore, a generating solution whose $E_{7(7)}$ orbit traces out the entire 56 dimensional space of extremal black holes must have at least five charges, of which one pair must be electric-magnetic duals [20, 21]. We are interested in finding candidate supergravity microstate solutions for such a generating black hole.

In the previous section we described a class of candidate microstates for five dimensional black rings and black holes in M-theory having a $U(1)$ isometry. These solutions carry three M2-brane charges and momentum along the $U(1)$ direction. Upon compactifying along this direction, these charges give rise to D2-branes and D0-branes in IIA string theory. To carry out the reduction to four dimensions we modify the solution in the previous section so that the $U(1)$ direction approaches a finite size at infinity. M5-branes can wrap on this circle leading to three D4-brane charges in IIA string theory. In addition, the now arbitrary NUT charge in the solution leads to D6-brane charge, giving in total eight charges.⁴ A similar procedure, of placing five dimensional solutions in Taub-Nut, has been used to relate five dimensional black holes and rings to four dimensional black holes [22, 23]. All of these charges arise from wrapped D6-branes with fluxes. Since D2s and D4s as well as D0s and D6s are electric-magnetic duals, these configurations also have the charges necessary for them to be smooth microstates associated to a finite area, extremal 1/8-BPS black hole.

3.1 Introducing Taub-NUT

The five dimensional solution in section 2 has a direction of $U(1)$ isometry. In order to reduce on this circle it must approach a finite size at infinity. We can accomplish this by adding a constant to H which effectively places the M2-branes in a Taub-NUT background:

$$H \rightarrow H + \delta H \quad ; \quad \delta H = \frac{4}{L^2}. \quad (3.1)$$

⁴Given an arbitrary NUT charge n_T , the asymptotic solution has topology S^3/Z_{n_T} . This allows us to define three Z_{n_T} valued 5-brane monopole charges. Once we reduce to IIA and the M-circle disappears, these give rise to regular integer 4-brane charges; shifts of these integer charges by n_T are associated with large gauge transformations of the B-field.

The asymptotic circumference of the circle is $2\pi L$. We allow the NUT charge (2.8) to be arbitrary, the full solution (2.1), (2.5) can then be interpreted in terms of M2-branes in a Taub-NUT background.

In order to ensure gauge invariance under transformations of the C-field it is necessary to also add constants δh^i to the h^i harmonic functions. In terms of these shifts it is convenient to define

$$\lambda_p^i = \frac{d_p^i}{n_p} - \frac{4\delta h^i}{\delta H} = \frac{d_p^i}{n_p} - L^2 \delta h^i, \tag{3.2}$$

and to re-write the the constant part of K (making sure there are no CTCs at infinity by bounding the last term) as:

$$\delta K = \frac{L^4}{4} \prod_i \delta h^i - \frac{L^2}{4} \sum_i \delta h^i + \frac{L}{2} \sin \alpha \tag{3.3}$$

To achieve a standard locally flat metric at infinity, we set the constant part of M_i to

$$\delta M_i = 1 - 2s_{ijk} \frac{\delta h^j \delta h^k}{\delta H}. \tag{3.4}$$

We can also set $k_0 \rightarrow 0$ at infinity by setting $\alpha = 0$. This is not necessary for getting a flat asymptotic metric, as we can always set this constant to zero by shifting the coordinate τ by a function of t . This adds another layer of unnecessary complication (see appendix B for the full construction), and without loss of generality we will assume that $\alpha = 0$. We do require, however, that the derivative of the angular momentum,

$$dk = d(k_0 f_a dx^a) + d(k_a dx^a) \tag{3.5}$$

falls off at infinity. This yields the constraint⁵

$$\sum_p \Psi_p = 0, \quad \Psi_p = \sum_i n_p \lambda_p^i - \frac{1}{L^2 n_p^2} \prod_i n_p \lambda_p^i. \tag{3.6}$$

In lifting back to five noncompact dimensions the shifts δh^i and δH are set to zero so that subleading terms in h^i and H determine the ratio

$$\lim_{r \rightarrow \infty} \frac{4h^i}{H} \rightarrow \frac{s^i}{n_T}. \tag{3.7}$$

Thus, in the decompactification limit (3.6) is tautological. The integrability condition on $k_a dx^a$ becomes

$$4\Psi_p + \sum_q \frac{\Gamma_{pq}}{r_{pq}} = 0, \quad p = 1 \dots N. \tag{3.8}$$

⁵When $\alpha \neq 0$ there is an additional term proportional to $\sin \alpha$ on the right hand side of (3.6). In the language of D=4, $\mathcal{N} = 2$ supersymmetry, this constraint relates the parameter α in (3.3) with the phase of the central charge, in a given gauge, of our 1/8-BPS solution (see appendix B). This connection between the angle α and the asymptotic velocity along the M-direction is typical for black rings (see [23]).

(Compare with (2.12) in the $L \rightarrow \infty$ decompactification limit.) Only $N - 1$ of these are independent if we take (3.6) into account. Specifying the angular momentum \vec{J}_L provides three more constraints (J_R will be related to the $D0$ -charge):

$$J_R = \sum_p n_p \prod_i \lambda_p^i \quad (3.9)$$

$$\vec{J}_L = -4 \sum_p \Psi_p \vec{x}_p = \frac{1}{2} \sum_{pq} \Gamma_{pq} \hat{r}_{pq}. \quad (3.10)$$

The second equality for \vec{J}_L used the constraint equation (3.8).

3.2 Reduction to IIA

We obtain a solution to IIA string theory by reducing along $y = (L/2)\tau$ (y has period $2\pi L$) in terms of dimensionless versions of the eight harmonic functions (M_i is already dimensionless)

$$M_0 = -HL^2/4, \quad K^0 = 4K/L, \quad K^i = Lh^i. \quad (3.11)$$

The reduction gives the metric

$$ds_{IIA}^2 = -J_4^{-1/2} (dt + k_a dx^a)^2 + J_4^{1/2} \left(ds_{\mathbf{R}^3}^2 + \sum_{i=1}^3 (-Z_i M_0)^{-1} ds_{T_i}^2 \right). \quad (3.12)$$

The radius of the compactification circle is related to the string length and coupling as

$$L = g_s l_s \quad (3.13)$$

The T_i are flat 2-tori with volume forms dV_i . The dilaton and form fields are

$$e^{2\Phi} = (J_4)^{3/2} (-Z_1 Z_2 Z_3 M_0^3)^{-1}, \quad B_2 = - \left(\frac{K^i}{M_0} + \frac{2k_0}{L Z_i} \right) dV_i \quad (3.14)$$

$$C_1 = \frac{L}{2} f_a dx^a - \frac{2M_0^2 k_0}{L J_4} (dt + k_a dx^a) \quad (3.15)$$

$$C_3 = \left[-Z_i^{-1} (dt + k_a dx^a) + 2\vec{a}_i - \left(\frac{K^i}{M_0} + \frac{2k_0}{L Z_i} \right) \frac{L}{2} f_a dx^a \right] \wedge dV_i. \quad (3.16)$$

J_4 is the quartic invariant of $E_{7(7)}$ constructed from the eight harmonic functions connected to four electric and magnetic ‘‘charges’’ [24]

$$J_4 = \frac{L^2}{4} \left((Z_1 Z_2 Z_3) H - k_0^2 H^2 \right) \quad (3.17)$$

$$\begin{aligned} &= M_0 K^0 (M_i K^i) + M_1 K^1 (M_2 K^2 + M_3 K^3) + M_2 K^2 M_3 K^3 \\ &\quad - \frac{1}{4} (M_\mu K^\mu)^2 - M_0 M_1 M_2 M_3 - K^0 K^1 K^2 K^3. \quad \mu \in 0 \dots 3 \end{aligned} \quad (3.18)$$

We have introduced a new radial variable $\rho = 2r/L$ so that the metric on the \mathbf{R}^3 piece takes the standard flat form.

For the further reduction on the T_i to four dimensions it is useful to define a shifted 3-form field that is invariant under shifts of coordinates in the y circle and the three 2-tori.

$$C'_3 = C_3 - B_2 \wedge C_1 = \left(\frac{-M_0}{J_4} (2K^i k_0/L - s^{ijk} Z_j Z_k/2) (dt + k_a dx^a) + 2\vec{a}_i \right) \wedge dV_i. \quad (3.19)$$

The quantized D -brane charges are

$$\begin{aligned} Q_0^{D6} &= \frac{L}{2} \sum_p (-n_p) = \frac{g_s l_s}{2} N_6, & Q_i^{D2} &= -\frac{1}{2L} \sum_p s_{ijk} \frac{d_p^j d_p^k}{2n_p} = -\frac{4G_4 V_i}{4\pi^2 g_s l_s^3} N_{i2}, \\ Q_{D0}^0 &= \frac{1}{2L^2} \sum_p \frac{d_p^1 d_p^2 d_p^3}{n_p^2} = \frac{4G_4}{g_s l_s} N_0, & Q_{D4}^i &= \frac{1}{2} \sum_p d_p^i = 2\pi^2 \frac{g_s l_s^3}{V_i} N_4^i. \end{aligned} \quad (3.20)$$

In our conventions the D0 and D4-branes are magnetic objects while the D2 and D6-branes are electric. While the charges defined here are quantized, they are not invariant under large gauge transformations of the B -field because of Chern-Simons terms in the supergravity Lagrangian. (See [25] for the difference between quantized and gauge-invariant charges.) In the four dimensional theory we will interpret large gauge transformations of the B -field as $SU(8)$ transformations inside $E_{7(7)}$. Our sign conventions are consistent with the Hodge dual relations $F_6 = *F_4, F_2 = *F_8$. Each of these charges is written as a sum over points p , and at each point we can interpret the charges as arising from a D6-brane with fluxes on it.

3.3 Reduction to four dimensions and special geometry

E_7 structure: Upon reduction to four dimensions, we obtain solutions to $\mathcal{N} = 8$ supergravity [26]. This theory has an $E_{7(7)}$ duality group. The three D2-brane charges and the D6-brane charge transform in an electric **28** representation of the maximal compact subgroup $SU(8)/\mathbf{Z}_2$, while the three D4-brane charges and the D0-brane transform in a magnetic **28**. Together, these charges transform in the **56** representation of $E_{7(7)}$; we can write combined charge vectors with eight of these charges turned on:

$$\Gamma_p = (Q_0^p, Q_i^p, Q_p^0, Q_p^i) = \left(\frac{-L}{2} n_p, \frac{-1}{4L} s_{ijk} \frac{d_p^j d_p^k}{n_p}, \frac{1}{2L^2} \frac{d_p^1 d_p^2 d_p^3}{n_p^2}, \frac{1}{2} d_p^i \right). \quad (3.21)$$

$$\Gamma = \sum_p \Gamma_p = (Q_0, Q_i, Q^0, Q^i) \quad (3.22)$$

(To avoid clutter we leave out the D0, D2 etc. notation in (3.20)). The symplectic E_7 invariant constructed from these charges is

$$\langle \Gamma_p, \Gamma_q \rangle = \frac{1}{2} (Q_p^0 Q_q^0 - Q_q^0 Q_p^0 + Q_p^i Q_q^i - Q_q^i Q_p^i) = \frac{\Gamma_{pq}}{8L} = G_N^{(4)} \gamma_{pq}, \quad (\gamma_{pq} \in \mathbf{Z}). \quad (3.23)$$

Similarly, we can think of our eight harmonic functions as part of a single harmonic function valued in the **56** of $E_{7(7)}$ written as (Γ_∞ denotes the constant terms):

$$\mathcal{H} = (M_0, M_i, K^0, K^i) = \Gamma_\infty + \sum_p \frac{\Gamma_p}{\rho_p}. \quad (3.24)$$

Then the 1-form in (2.9) which gives rise to the angular momentum in the solution satisfies

$$\star_3 d(k_a dx^a) = \star_3 d\vec{k} = \langle d\mathcal{H}, \mathcal{H} \rangle. \quad (3.25)$$

At this juncture, we see that the reduction of our M-theory solutions down to $\mathcal{N} = 8$ supergravity in four dimensions yields a framework similar to the $\mathcal{N} = 2$ setup in [27–29]. In fact, our theory with eight vector fields and six scalars corresponds to a truncation of the $\mathcal{N} = 8$ theory to the famous STU $\mathcal{N} = 2$ model [30, 35] which corresponds to the symmetric coset space $[\text{SL}(2, R)/\text{U}(1)]^3$.

Explicit solution and special geometry: The four-dimensional metric is the obvious truncation of (3.12)

$$ds_4^2 = -J_4^{-1/2} (dt + k_a dx^a)^2 + J_4^{1/2} ds_{\mathbf{R}^3}^2. \quad (3.26)$$

The four dimensional dilaton is constant, because the volume of the three 2-tori in (3.12) cancels the spatial dependence of the ten dimensional dilaton in (3.14). The solution has three complex scalar fields coming from the complexified Kahler moduli of the three T_i ($\phi^i = i\text{Vol}^i + B_2^i$). In terms of the explicit ten dimensional solution (3.12), (3.14), the scalars are

$$\phi^i = i \frac{-2J_4^{1/2}}{2M_0M_i - s_{ijk} K^j K^k} - \left(\frac{2M_i K^i + M_0 K^0 - \sum_j M_j K^j}{2M_0M_i - s_{ijk} K^j K^k} \right), \quad (3.27)$$

$$= \left(\frac{\partial J_4^{\frac{1}{2}}}{\partial M_i} - \frac{i}{2} K^i \right) / \left(\frac{\partial J_4^{\frac{1}{2}}}{\partial K^0} + \frac{i}{2} M_0 \right). \quad (3.28)$$

In the first line there is no sum on i . In the second line the scalars have been written entirely in terms of the J_4 invariant and individual harmonic functions transforming in an eight-dimensional subspace of the **56** of $E_{7(7)}$. These six scalars are part of the **70** scalars of the $E_{7(7)}/\text{SU}(8)$ coset. Finally, the C_3 field and C_1 field give $3 + 1$ vectors (the above three vector multiplets plus the graviphoton) which transform, along with their duals, in the same subspace of the **56** representation as the harmonic functions. The potentials and their duals can be summarized in a single vector as

$$\vec{\mathcal{A}} = \left(\vec{\mathcal{A}}_0, \vec{\mathcal{A}}_i, \vec{\mathcal{A}}^0, \vec{\mathcal{A}}^i \right), \quad (3.29)$$

with components from reducing C_3' and C_1 :

$$\vec{\mathcal{A}}^i = -\frac{1}{J_4} \frac{\partial J_4}{\partial M_i} (dt + k_a dx^a) + 2\vec{a}^i, \quad \vec{\mathcal{A}}_0 = +\frac{1}{J_4} \frac{\partial J_4}{\partial K^0} (dt + k_a dx^a) + 2\vec{a}_0, \quad (3.30)$$

and their duals:

$$\vec{\mathcal{A}}_i = +\frac{1}{J_4} \frac{\partial J_4}{\partial K^i} (dt + k_a dx^a) + 2\vec{a}_i, \quad \vec{\mathcal{A}}^0 = -\frac{1}{J_4} \frac{\partial J_4}{\partial M_0} (dt + k_a dx^a) + 2\vec{a}^0. \quad (3.31)$$

The magnetic parts of these potentials are concisely written as:

$$d\vec{a} = \left(d\vec{a}_0, d\vec{a}_i, d\vec{a}^0, d\vec{a}^i \right) = -\star_3 d\mathcal{H}. \quad (3.32)$$

$E_{7(7)}$ and more general solutions: The solutions presented above have four vector fields and six scalars, corresponding to a truncation of the the full $\mathcal{N} = 8$ theory to the $\mathcal{N} = 2$ STU model [35]. More generic four dimensional solutions with arbitrary charges can be obtained by generalizing the above, allowing the harmonic functions (3.24) as well as the individual charge vectors (3.21) to occupy the full **56** of $E_{7(7)}$. Typically, however, the most general configurations of this type will break supersymmetry. Since we are interested in 1/8-BPS states, our charges always need to line up with a preferred $\mathcal{N} = 2$ subalgebra, corresponding to the reduction of $E_{7(7)}$ to $SO^*(12) \times SU(2)_R$ (for more details see [31–33]). For such configurations, the appropriate charge subspace for supersymmetry yields vectors (and harmonic functions) which transform in the real **32** spinor representation of $SO^*(12)$, accompanied by thirty non-trivial scalars. The remaining forty scalars of the $\mathcal{N} = 8$ theory appear as hypermultiplet scalars with respect to the $N = 2$ truncation, parameterizing the coset $E_{6(2)}/SU(2)_R \times SU(6)$. Their values stay fixed throughout our solutions, and are generally constrained by the alignment of the subgroup $SO^*(12) \times SU(2)_R$ inside $E_{7(7)}$.

Generalized constraint and coarse graining: For general charge vectors Γ_p the constraint equation (3.8) can be appropriately generalized by requiring integrability (3.25) as before. Then, in terms of a symplectic product of the charges and the asymptotic values of the harmonic functions (3.24),

$$\langle \Gamma_p, \Gamma_\infty \rangle + \sum_q \frac{\langle \Gamma_p, \Gamma_q \rangle}{\rho_{pq}} = 0. \tag{3.33}$$

The first term automatically provides an expression for the Ψ_p 's and summing these equations provides the first of the two constraints on Γ_∞ ,

$$\langle \Gamma, \Gamma_\infty \rangle = 0, \quad I_4(\Gamma_\infty) = 1, \tag{3.34}$$

which tell us that the asymptotic scalars sit in the appropriate coset.

In the form above, the constraint equation also gives us an insight into the behavior of the solution if we “coarse-grain” over a collection of charges by collecting them into a single pole. For example, let us partition our poles into sets \mathcal{P} containing poles separated by distances much smaller than some reference scale, Λ . Then we can define a coarse-grained solution by assigning the total charge of each cluster \mathcal{P} to the mean location of the poles in the cluster:

$$\Gamma_{\mathcal{P}} = \sum_{p \in \mathcal{P}} \Gamma_p, \quad \vec{x}_{\mathcal{P}} \sim \langle \vec{x}_p \rangle. \tag{3.35}$$

The constraints on the coarsened solution are then:

$$\langle \Gamma_{\mathcal{P}}, \Gamma_\infty \rangle + \sum_{\mathcal{Q}} \frac{\langle \Gamma_{\mathcal{P}}, \Gamma_{\mathcal{Q}} \rangle}{\rho_{\mathcal{P}\mathcal{Q}}} \ll \Lambda. \tag{3.36}$$

An important feature of this coarse-graining is that if a cluster of poles \mathcal{P} has the property that $\langle \Gamma_{\mathcal{P}}, \Gamma_\infty \rangle = 0$, then this cluster can be placed at an infinite distance from the rest of the poles, distinguishing our supergravity solution as one generated by at least two separate bound states.

The maximally coarse-grained solution replaces the detailed microstate by a solution with a single pole carrying the total charge vector of the spacetime. Below we will show that such single-pole solutions are black holes of the $\mathcal{N} = 8$ supergravity and will in general have a finite horizon area.

3.4 Relation to finite area black holes

We would like to understand how the solutions described above can be seen as candidate microstates for the extremal black holes of $\mathcal{N} = 8$ supergravity. Far away from all the poles, the harmonic function \mathcal{H} can be well approximated by the single center function

$$\bar{\mathcal{H}} = \Gamma_\infty + \frac{\Gamma}{\rho} \tag{3.37}$$

where Γ is the total charge vector defined in (3.22). Plugging this simplified function into our formulism yields a metric with no angular momentum⁶ and

$$J_4(\rho) = I_4(\Gamma)\rho^{-4}, \tag{3.38}$$

where $I_4(\Gamma)$ comes from expression (3.17) for J_4 after we substitute in the appropriate charge for each harmonic function. If $I_4(\Gamma) > 0$, this is simply the spherically symmetric metric of a 1/8-BPS black hole with total charge vector Γ and horizon area given by

$$A = 2\pi\sqrt{I_4(\Gamma)}. \tag{3.39}$$

If $I_4(\Gamma) < 0$ (the null case will become clear later), we have taken our coarse-graining procedure too far, and need to retreat back until we only have centers with $I_4(\Gamma_{\mathcal{P}}) \geq 0$. The astute reader can recognize I_4 as Cartan's quartic E_7 invariant [18, 19, 5, 34]. In terms of the antisymmetric central charge matrix $z_{ij} = x_{ij} + iy_{ij}$ of $\mathcal{N} = 8$ supergravity, this invariant is written as

$$4I_4 = \frac{x^{ij}y_{ij}x^{kl}y_{kl} - x^{ij}y_{kj}x^{kl}y_{li} - \frac{1}{96}(\epsilon^{ijklmnop}x_{ij}x_{kl}x_{mn}x_{op} + \epsilon_{ijklmnop}y^{ij}y^{kl}y^{mn}y^{op})}{4}. \tag{3.40}$$

Here indices are raised and lowered by δ_j^i . In our conventions⁷

$$x^{12} = Q^1 \quad ; \quad x^{34} = Q^2 \quad ; \quad x^{56} = Q^3 \quad ; \quad x^{78} = Q^0 \tag{3.41}$$

$$y_{12} = Q_1 \quad ; \quad y_{34} = Q_2 \quad ; \quad y_{56} = Q_3 \quad ; \quad y_{78} = Q_0 \tag{3.42}$$

For a solution with only the D2 (D4) and D6 (D0) brane charges, the expected horizon area of the associated black hole is

$$\text{D2 - D2 - D2 - D6 area :} \quad A = 2\pi\sqrt{-Q_0 \prod_i Q_i} \tag{3.43}$$

$$\text{D4 - D4 - D4 - D0 area :} \quad A = 2\pi\sqrt{-Q^0 \prod_i Q^i} \tag{3.44}$$

⁶Non-zero angular momentum is contingent on keeping at least a dipole moment when approximating \mathcal{H} .

⁷The J_4 invariant in [19] differs by a factor of 4 from the one here, and charge conventions there are related to the present ones by a factor of $\sqrt{2}$ and sign flip of the D6-brane charge.

The sign under the square root indicates that the D6 (D0) must be appropriately oriented relative to the D2 (D4) branes to preserve supersymmetry [21]. If electric and magnetic dual objects are present, other terms in I_4 will also contribute. The space of all extremal finite area black holes in $\mathcal{N} = 8$ supergravity can be generated by $E_{7(7)}$ transformations of generating configurations with five charges containing one electric-magnetic pair [31, 21, 19]. An example generating configuration contains three D2-brane, D6-brane and D0-brane charges. Such charge vectors are included in our analysis and thus the solutions described earlier in this section provide candidate microstates for the generating black holes of $\mathcal{N} = 8$ supergravity.

It is interesting to ask whether a charge vector of the form (3.21) associated to a single pole in the solution could have given rise to a finite horizon area by itself. Knowing the leading behavior of the functions Z_i, k_0 and H it easy to show that

$$\lim_{\vec{x} \rightarrow \vec{x}_p} J_4 \propto \rho_p^{-1} \quad \Rightarrow \quad I_4(\Gamma_p) = 0. \quad (3.45)$$

We can also check that the invariant $I_4(\Gamma_p)$ vanishes in this case by using the conventions (3.41), (3.42) and the single pole charge vector as given in (3.21). Thus, a black hole carrying the charges of a single pole in our solutions (3.21) would have vanishing horizon area and entropy. In fact, the growth of J_4 near one of our “primitive” poles tells us even more. In [21] the authors distinguish four kinds of BPS black holes in $\mathcal{N} = 8$ supergravity. One can find examples for each of these by once again looking at configurations with just D2 and D6-brane charge:

$$\begin{aligned} D2 - D2 - D2 - D6 \neq 0 : & \quad 1/8 \text{ BPS}, \quad I_4 > 0, \quad J_4 \propto \rho^{-4}, \\ D2 - D2 - D6 \neq 0 : & \quad 1/8 \text{ BPS}, \quad I_4 = 0, \quad J_4 \propto \rho^{-3}, \\ D2 - D6 \neq 0 : & \quad 1/4 \text{ BPS}, \quad I_4 = 0, \quad \partial I_4 = 0, \quad J_4 \propto \rho^{-2}, \\ D6 \neq 0 : & \quad 1/2 \text{ BPS}, \quad I_4 = 0, \quad \partial I_4 = 0, \quad \partial \partial|_{\text{Adj}} I_4 = 0, \quad J_4 \propto \rho^{-1}. \end{aligned} \quad (3.46)$$

The notation with ∂ 's is impressionistic, see [21] for more detail. Thus, the rate of growth of J_4 yields a simple U-duality invariant method for determining how much supersymmetry is preserved by a given black hole or pole in a multi-pole configuration.

It has been shown on general grounds that to be associated to a finite horizon area, the charge vector of a black hole in $\mathcal{N} = 8$ supergravity can preserve at most four supercharges — such states, including solutions with general 2-brane and 6-brane charges are 1/8-BPS. By contrast, the classification above tells us that charge vectors of individual poles in our solutions are all 1/2-BPS. Another simple way of seeing this is to note that the charges in (3.21) are consistent with having $-n_p$ 6-branes with worldvolume fluxes turned on in the 12, 34, and 56 directions. The Chern-Simons couplings on the brane would then precisely reproduce the 4-brane, 2-brane and 0-brane charges given in (3.21). Such states of 6-branes with fluxes are known to be 1/2-BPS, they are T-dual to IIB 3-branes at angles on a T^6 .

Previous work has shown how certain supersymmetric black holes could be thought of as single center marginal bound states of 1/2 BPS objects later understood to be D-branes [35–38]. Indeed the four classes of solutions in (3.46) correspond precisely to the

four types of axion charges appearing in [35–38]. In our work, the component 1/2 BPS states are spatially separated and are held together in a true bound state because of their nonzero intersection numbers.

$E_{7(7)}$ transformations of microstates: In general, we can use an $E_{7(7)}$ transformation to take any finite area 1/8 BPS black hole of $\mathcal{N} = 8$ SUGRA with all **56** charges to one with only the eight charges of the STU model [31, 35], with scalars in $[\text{SL}(2, R)/\text{SO}(2)]^3$. As we mentioned above, we can further use the three compact $\text{SO}(2)$'s of the STU models to eliminate⁸ three more of the charges to get, for example, a model with just D2-D2-D2-D6-D0 charges. Recall, however, that the proposed microstates for such a black hole will contain a large number of poles with individual charge vectors. Even if the overall black hole charges fall within the STU sector, the charges of the component poles in the underlying microstates will not be restricted in this way since there is not sufficient symmetry to rotate each pole individually; they will typically all lie in different subspaces of the spinor of $\text{SO}^*(12)$ [33].

3.5 Summary and proposal

In this section, we reduced a class of smooth candidate microstate solutions for supersymmetric five dimensional black holes and rings to microstates for four dimensional black holes with eight charges. These spacetimes were written as multi-center solutions in which each center served as a 1/2-BPS “atom” for building up the full configuration. The bound state nature of the overall solution was maintained by the mutual non-locality of the charges which led to constraints on their relative positions. This motivates a conjecture:

Every supersymmetric 4D black hole of finite area, preserving 4 supercharges, can be split up into microstates made of primitive 1/2-BPS “atoms”, each of which preserves 16 supercharges. In order to describe a bound state, these atoms should consist of mutually non-local charges.

The next section provides evidence for this picture.

4. From spacetime foam to D-branes

In [39–43] the entropy of four dimensional black holes of finite area was accounted for in terms of D-brane bound state degeneracies. The basic strategy was to use D-branes to count the states in the $g_s \rightarrow 0$ limit and then extrapolate back to stronger coupling using supersymmetry. As such the microstructure of the black hole arose from the many degenerate ground states of the D-branes wrapped on the internal space, in our case T^6 . The picture offered above suggests instead that the entropy of the black hole arises from structure in the non-compact four dimensions of spacetime. This is more along the lines of the proposals of Mathur and collaborators [44–46]. In this section we demonstrate how these two approaches to black hole entropy could be reconciled. The classical supergravity

⁸Actually, the full quantum theory has a more restricted U-duality group which only allows us to reduce the D4-brane number charges to be in the interval $0 \dots (N_6 - 1)$, this is the IIA manifestation of the fact that M-theory on Taub-NUT has Z_{N_6} torsion charges for M5-branes.

solutions that we have found flow at weak coupling to systems of intersecting branes of the sort originally used in [39–43] to count black hole microstates. This suggests that the ground states of such coincident branes at vanishing g_s flow at weak g_s to the four dimensional supergravity solutions that we have described and at strong g_s to a spacetime foam in M-theory. Evidence for this picture exists in the analysis of flows of BPS brane bound states discussed by Denef [13].

4.1 A scaling relation

The locations of the centers in each four dimensional geometry in section 3 are determined by the constraint equation (3.8). We can examine how these solutions change as

$$g_s \rightarrow \beta g_s \tag{4.1}$$

while we hold the volume of the torus fixed in string units

$$\frac{\prod_i V_i}{l_s^6} = \text{fixed} . \tag{4.2}$$

The quantized charges of the branes, constructed from the integers n_p and m_p^i (2.16) are held fixed, and thus the physical charges (3.20) scale in powers of β . Putting everything together, under the rescaling (4.1), the constraint equation (3.8) becomes

$$4\Psi_p\beta + \beta^3 \sum_q \frac{\Gamma_{pq}}{r_{pq}} = 0 . \tag{4.3}$$

Thus, given a set of separations $\{r_{pq}\}$ that solve the constraint equation for a string coupling g_s , the set of separations $\{\beta^2 r_{pq}\}$ solves the constraints for a coupling βg_s . The physical separations are

$$\rho_{pq} = \frac{2r_{pq}}{L} \tag{4.4}$$

and these scale linearly with g_s :

$$\rho_{pq} \rightarrow \beta \rho_{pq} . \tag{4.5}$$

In order to continue to satisfy the no-CTC condition (2.13) we must also scale the coordinate ρ as

$$\rho \rightarrow \beta \rho . \tag{4.6}$$

This scaling has far reaching consequences. As we go to weaker coupling, the branes move closer together in string units. At some value of the coupling the branes will move within a string length of each other and the appropriate description is in terms of the open strings on the branes.

To estimate when the open string picture becomes valid, first consider the two-center case. Define the dimensionless quantity

$$\psi_p = (g_s l_s)^{-1} \Psi_p . \tag{4.7}$$

The branes will be much closer than a string length apart when⁹

$$g_s \ll \frac{1}{32\pi^6} \frac{\prod_i V_i}{l_s^6} \left| \frac{\psi_1}{\gamma_{12}} \right|. \quad (4.8)$$

When all brane separations are roughly of the same order we can estimate that all pairs of branes are closer than the string length when

$$g_s \ll \frac{1}{32\pi^6} \frac{\prod_i V_i}{l_s^6} \left| \frac{\psi_p}{\gamma_{pq}} \right| \quad \forall \psi_p, \gamma_{pq}. \quad (4.9)$$

For a general bound state, there will always be a value of g_s small enough that all branes are separated by distances smaller than the string length. For such small g_s the supergravity solutions described in section 3 are better described in terms of open string degrees of freedom coming from the D-branes.

Going the other way, as g_s increases, the intervals between the IIA D-branes in the solution increase until, at large coupling the IIA description is no longer valid. At that point we move to an M-theory description with the multi-center D-brane solution replaced by a network of two-cycles (“bubbles”) we call spacetime foam. The new reference length becomes the 11-dimensional Planck length; the size of the asymptotic circle in Planck units now becomes a dimensionless modulus like all the others. Recently, a similar flow from D-branes to “spacetime foam” has been noted in the topological string [14].

4.2 The open string picture

When D-branes are much closer than a string length, an open string description is appropriate. The vacua of the brane system can be analyzed just in terms of the low-energy field theory on the D-branes if the massive string excitations can be integrated out. This is the case when all the brane separations are less than l_s and when all the charge vectors are sufficiently closely aligned. Taking these conditions to be satisfied, we will describe how the supergravity states in section 3 appear as vacua of a D-brane gauge theory when g_s is sufficiently small. In this section, we will set $l_s = 1$.

First consider a single center with charge vector $\Gamma_p = N_p \hat{\Gamma}_p$ where N_p is the greatest common divisor of the charges appearing in Γ_p , and $\hat{\Gamma}_p$ is thus a *primitive* charge vector. This represents a stack of N_p D-branes wrapping T^6 . When the torus is small (as we take it to be), the low energy physics is obtained by dimensionally reducing the D-brane gauged field theory to a gauged quantum mechanics. Thus the latter has the field content of dimensionally reduced $\mathcal{N} = 4, d = 4$, $U(N_p)$ super-Yang-Mills theory. However, since the interactions between different stacks of branes will only preserve four supercharges, it is convenient to decompose into multiplets of $\mathcal{N} = 4, d = 1$ Yang-Mills, obtained via dimensional reduction of an $\mathcal{N} = 1, d = 4$ theory. This gives one vector multiplet and three adjoint chiral multiplets. The vector multiplet contains three real, adjoint scalars – these parametrize the positions of the stack of branes in the non-compact space. The three

⁹Recall in the two-pole case, with $\alpha = 0$, the constraint (3.6) sets $\Psi_1 = -\Psi_2$.

complex adjoints parametrize Wilson lines on the 6-branes (or positions within the torus after T-duality).¹⁰

In addition, stacks of branes with integer intersection number $\gamma_{pq} \neq 0$ (3.23) give rise to $|\gamma_{pq}|$ chiral fields ϕ_{pq} charged in the bifundamental of $U(N_p) \times U(N_q)$ [47–49]. If $\gamma_{pq} = 0$, i.e. if the charge vectors Γ_p and Γ_q are mutually local, then bifundamentals for these branes only appear in chiral and anti-chiral pairs. There is a Higgs term coupling these bifundamentals to the massless adjoint multiplets on each of the two branes. In a generic bound state, however, mutually local charge vector pairs will occur rarely. Hence we will focus on situations where $\gamma_{pq} \neq 0 \forall p, q$.

Thus, the low-energy field theory describing the system of branes at small g_s is a $d = 1$, $\mathcal{N} = 4$ quiver quantum mechanics constructed as follows:

1. For each stack of D6-branes at point p with charge vector Γ_p we associate a $U(N_p)$ gauge group and vector multiplet. Here $\Gamma_p = N_p \hat{\Gamma}_p$ where N_p is the GCD of the charges in Γ_p and $\hat{\Gamma}_p$ is *primitive*. The three real, adjoint scalars (\vec{x}) in the vector multiplet correspond to location in the \mathbf{R}^3 transverse to the branes.
2. The number of bifundamental chiral fields ϕ_{pq} (transforming in the (\bar{N}_p, N_q) of $U(N_p) \times U(N_q)$) between branes p and q is given by the intersection number γ_{pq} .
3. Since each charge vector Γ_p corresponds to a 1/2-BPS state, we will also get at each node the remainder of an $\mathcal{N} = 4$ vector multiplet: three adjoint chiral multiplets. These will mostly play a spectator role in our considerations.

The general Lagrangian for the vector multiplet, the bifundamental chiral multiplet and their couplings is given in appendix C of [13]. The terms arising from the additional chiral adjoints can be determined by dimensional reduction of the $\mathcal{N} = 4, d = 4$ Lagrangian. For determining the vacua and phases of the theory, we need to know how these additional fields contribute to the D-term and F-term equations and to the masses of the bifundamentals. The FI-term in the Lagrangian takes the form [13]

$$L_{\text{FI}} = \sum_p \mathcal{F}_p \text{Tr} D_p, \tag{4.10}$$

where D_p is the auxiliary adjoint field in the $U(N_p)$ vector multiplet. \mathcal{F}_p is linear in the charges and depends on closed string field values. It only couples to the D-term for the diagonal $U(1)$ of $U(N_p)$. This is consistent with the notion that if we just slightly separate our stack into two stacks, $U(N_p) \rightarrow U(N_{p1}) \times U(N_{p2})$, then $\mathcal{F}_p = \mathcal{F}_{p1} = \mathcal{F}_{p2}$.

Since the adjoint chirals are neutral under the diagonal center-of-mass $U(1)$ they cannot couple to the corresponding D-term and hence to the D-term equation which will most

¹⁰Under T-duality of the torus, the 6-branes we are describing can be transformed to a system of wrapped 3-branes. The worldvolume theory of these branes consists of a $\mathcal{N} = 4, d = 4$ vector multiplet which decomposes into a vector multiplet of $\mathcal{N} = 1, d = 4$ Yang-Mills and three adjoint chiral multiplets. The real parts of these adjoint scalars parameterize positions in the non-compact space, while the imaginary parts describe positions on the torus. Upon dimensional reduction, the real parts of the $d = 4$ adjoints become the three real scalars in the $\mathcal{N} = 4, d = 1$ vector multiplet. The imaginary parts of the $d = 4$ adjoints pair up with the Wilson lines on the 3-brane to build the three adjoint chiral multiplets of the $d = 1$ theory.

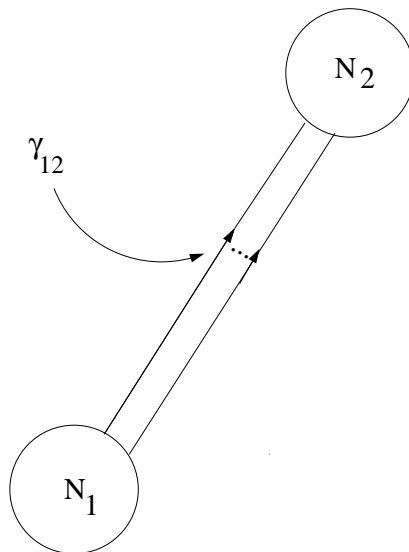


Figure 1: A sample quiver with two nodes.

interest us; they only contribute to the other $N_p - 1$ equations coming from the $SU(N_p)$ D-terms. What is more, up to at least quadratic order, the adjoint scalars do not have Higgs couplings to chiral bifundamentals which are not paired to anti-chiral ones. To see this, consider a pair of branes with bifundamentals running between them. The pair can be T-dualized to give two 3-branes at angles in IIB string theory. In this context, the expectation values of the adjoint scalars in the quantum mechanics Lagrangian parameterize the positions of the 3-branes on T^6 and the Wilson lines in the branes. Neither of these affect either the number of intersections between the branes, or the spectrum of strings localized at the intersection points. Hence there are no Higgs couplings up to at least quadratic order between the adjoint and the bifundamental chiral fields. Finally, the potential for the adjoint chiral multiplets is inherited from $\mathcal{N} = 4, d = 4$ Yang-Mills and simply forces the adjoints to commute on the vacuum manifold.

Hence, for the purpose of studying the phases and vacua of our quiver quantum mechanics, we can largely ignore the adjoint chiral multiplets. Fortunately, the remaining problem is identical to the one studied by Denef in [13] and in section 4.3 we simply adapt his analysis to our situation. The vacuum manifold of the quiver quantum mechanics can have a Coulomb branch in which the vector multiplet scalars (\vec{x}_p) have expectation values, and a Higgs branch in which the chiral multiplet scalars ϕ_{pq} are given VEVs. We will study each in turn and discuss how the supergravity states in section 3 appear in the Coulomb branch and how they can flow into the Higgs branch as $g_s \rightarrow 0$.

An example black hole microstate: Before proceeding it is worthwhile to give an example showing that quivers exist with charges appropriate for being microstates of black holes with finite area. Since each D-brane center is 1/2-BPS we will require a minimum of three nodes in the quiver (see section 3.4). The conditions to be satisfied are:

1. All charges and charge vectors must be appropriately quantized (1/2-BPS in the case

of individual centers).

2. $J_4(\Gamma) > 0$ where Γ is the total charge vector so that the collection has the charges to be a candidate microstate for a finite horizon area black hole.
3. There exist solutions of the constraint equations (3.33) that also satisfy the triangle inequalities for brane positions. This is the only one of our conditions which depends on the asymptotic moduli.

Working in units such that $e^i = 1, \forall i$ and $L = 1$ (this sets $G_N^{(4)} = 1/8$), the charge vectors can be written $\Gamma_p = \frac{1}{2}(N_0^p, N_i^p, N_p^0, N_p^i), i = 1 \dots 3$ (3.21), where as before the N charges are quantized. We will also specialize to diagonal 2 and 4-brane charges (i.e. $N_p^i = N_p, N_i^p = N^p, \forall i$). An example quiver meeting our requirements arises from the charge vectors

$$\Gamma_1 = \frac{1}{2}(6, 0, 0, 0), \quad \Gamma_2 = \frac{1}{2}(-1, -1, 1, 1), \quad \Gamma_3 = \frac{1}{2}(-2, -2, -2, -2). \quad (4.11)$$

The total charge vector and J_4 invariant are

$$\Gamma = \sum_p \Gamma_p = \frac{1}{2}(3, -3, -1, -1) \quad ; \quad J_4(\Gamma) = 71/16 = 71 \cdot \left(2G_N^{(4)}\right)^2 > 0. \quad (4.12)$$

The intersection numbers

$$\gamma_{12} = -6, \quad \gamma_{23} = -16, \quad \gamma_{31} = -12, \quad (4.13)$$

indicate that we have closed loops in the quiver.

For such a closed loop there exists a simple “scaling solution” [13] where the centers converge on each other with separations limiting to a set congruent with the triangle made up of the γ_{pq} 's. This is independent of the value of the asymptotic moduli set by our choice of Γ_∞ .

4.3 Gauge theory analysis

The results of [13] are expressed in the language of $\mathcal{N} = 2$ supergravity, in terms of the central charge associated to each brane in the quiver. In appendix A the central charge of the p^{th} brane is shown to be

$$Z_p = \frac{L}{2}n_p - \frac{1}{2L} \sum_{i,j,k} \frac{s_{ijk}}{2} n_p \lambda_p^j \lambda_p^k + \frac{i}{2} \Psi_p. \quad (4.14)$$

In terms of Z_p the mass of the brane is

$$m_p = \frac{|Z_p|}{4G_4} \quad (4.15)$$

and we can write

$$Z_p = |Z_p| e^{i\alpha_p} \quad ; \quad \sin \alpha_p = \frac{\text{Im} Z_p}{|Z_p|}. \quad (4.16)$$

The total central charge is

$$Z = \sum_p Z_p \quad ; \quad Z \equiv |Z| e^{i\alpha} \quad ; \quad \sin \alpha = \frac{\text{Im} Z}{|Z|}. \quad (4.17)$$

In our solutions the constraint (3.6) leads to $\alpha = 0$.¹¹ In the field theory analysis below the parameter

$$\theta_p = \frac{\text{Im}(e^{-i\alpha} Z_p)}{4G_4} = |Z_p| \frac{\sin(\alpha_p - \alpha)}{4G_4} \approx |Z_p| \frac{(\alpha_p - \alpha)}{4G_4} \quad (4.18)$$

will play a role. The last approximate equality holds when the phases of all the branes are nearly equal, as required for a field theory analysis to be valid. Since we are working with solutions with $\alpha = 0$, this means that all the α_p are small also. Thus

$$\sin \alpha_p \approx \alpha_p = \frac{\text{Im} Z_p}{|Z_p|}. \quad (4.19)$$

Putting everything together,

$$\theta_p \approx \frac{\Psi_p}{8G_4}. \quad (4.20)$$

Armed with these quantities we can adapt Denef's results [13] to our setting.

We will consider quivers that do not contain closed loops. This ensures that the bifundamental chiral multiplets do not have a superpotential and hence analyzing the D-term equations is sufficient to determine the vacuum structure. Also, for simplicity, we will begin by taking $N_p = 1$ (a U(1) gauge theory) at each quiver node. The non-Abelian case will follow from this. For an abelian quiver ($N_p = 1, \forall p$) without closed loops the relevant part of the bosonic effective Lagrangian is [13]:

$$L_{\text{eff}} = \sum_p \left[\frac{m_p}{2} D_p^2 - \theta_p D_p \right] + \sum_{p < q} \sum_{a=1}^{|\gamma_{pq}|} \left[|F_{pq}^a|^2 - (|\vec{x}_p - \vec{x}_q|^2 + (D_p - D_q)(-1)^{s_{pq}}) |\phi_{pq}^a|^2 \right]. \quad (4.21)$$

Here \vec{x}_p and D_p are the three scalar fields and the auxiliary field of the p^{th} vector multiplet, ϕ_{pq}^a are the $|\gamma_{pq}|$ bifundamentals charged under $U(1)_p \times U(1)_q$, F_{pq}^a are the corresponding auxiliary fields, and

$$s_{pq} = \text{sign}(\gamma_{pq}). \quad (4.22)$$

We have left out the standard kinetic terms and fermionic pieces. If some of the $N_p > 1$, additional commutator terms and appropriate traces are required.

Coulomb branch: When the vector multiplet scalars are given an expectation value, the bifundamental fields between the branes at p and at q have a mass

$$\left(m_{pq}^\phi \right)^2 = |\vec{x}_p - \vec{x}_q|^2 + (D_p - D_q)(-1)^{s_{pq}}. \quad (4.23)$$

The fermionic partner of ϕ_{pq}^a has a mass (see appendix C of [13])

$$\left(m_{pq}^\psi \right)^2 = |\vec{x}_p - \vec{x}_q|^2. \quad (4.24)$$

Thus the fields in the chiral multiplet can be integrated out to give an effective Lagrangian for the fields in the vector multiplet. We are particularly interested in terms that are

¹¹A non-zero overall phase is easily restored by including solutions that carry a velocity along the Taub-Nut direction, i.e. by allowing a total charge vector Γ such that $\tilde{\alpha} \neq 0$.

linear in D_p since these make up the Fayet-Iliopoulos parameter whose vanishing gives the condition for supersymmetry. Fortunately, a non-renormalization theorem guarantees that this will be exact at one-loop. The bosonic effective Lagrangian for the vector multiplet at one-loop order in the chiral multiplet is

$$L_{\text{eff}}^V = \sum_p \left[\frac{m_p}{2} D_p^2 - \theta_p D_p \right] + \sum_{p < q} \sum_{a=1}^{|\gamma_{pq}|} \ln \det \left[\frac{-\partial_t^2 + |\vec{x}_p - \vec{x}_q|^2}{-\partial_t^2 + |\vec{x}_p - \vec{x}_q|^2 + (D_p - D_q)(-1)^{s_{pq}}} \right]. \quad (4.25)$$

The determinants are standard and give

$$L_{\text{eff}}^V = \sum_p \left[\frac{m_p}{2} D_p^2 - \theta_p D_p \right] + \sum_{p < q} |\gamma_{pq}| \left(|\vec{x}_p - \vec{x}_q| - \sqrt{|\vec{x}_p - \vec{x}_q|^2 + (D_p - D_q)(-1)^{s_{pq}}} \right). \quad (4.26)$$

The D-term equation $(\partial L_{\text{eff}}^V / \partial D_p)|_{D_p=0} = 0$ gives

$$\sum_q \frac{\gamma_{pq}}{2 |\vec{x}_p - \vec{x}_q|} = -\theta_p, \quad (4.27)$$

which, when combined with (4.20), gives the supersymmetry conditions

$$\sum_q \frac{\gamma_{pq}}{|\vec{x}_p - \vec{x}_q|} = -\frac{1}{4G_4} \Psi_p. \quad (4.28)$$

The solutions to this equation form the moduli space of supersymmetric vacua in the Coulomb branch. Now recall that our supergravity solutions satisfy a constraint equation $\sum_p \Gamma_{pq}/r_{pq} = -4\Psi_p$ (3.8). Recalling the relation (3.23) between Γ_{pq} and the integer intersection numbers γ_{pq} , as well as the relation (4.4) between r_{pq} and the physical separations ρ_{pq} , the supergravity constraint becomes

$$\sum_p \frac{\gamma_{pq}}{\rho_{pq}} = -\frac{1}{4G_4} \Psi_p. \quad (4.29)$$

It is beautiful that (4.28) and (4.29) match identically. This precise match teaches us that, following the scaling relation in section 4.1, as g_s decreases each supergravity solution in section 3 flows smoothly into a corresponding solution in the gauge theory Coulomb branch.¹²

Higgs branch: The scaling relation in section 4.1 applies equally to the Coulomb branch equation (4.28). Hence, after our states have flowed into the Coulomb branch, a reduction of g_s will cause a further decrease in $|\vec{x}_p - \vec{x}_q|$, and with it the mass of the chiral multiplet. If this mass becomes too small, the field cannot be integrated out. To study when this happens, we can eliminate the auxiliary field D_p from (4.21) via its equation of motion and find the mass of ϕ_{pq}^a :

$$\begin{aligned} (m_{pq}^\phi)^2 &= |\vec{x}_p - \vec{x}_q|^2 + \left(\frac{\theta_p}{m_p} - \frac{\theta_q}{m_q} \right) (-1)^{s_{pq}} \\ &= |\vec{x}_p - \vec{x}_q|^2 + 4G_4(\alpha_p - \alpha_q) (-1)^{s_{pq}}. \end{aligned} \quad (4.30)$$

¹²Strictly speaking, in situations where some brane separations are much larger than others, parts of the solution can flow into the Coulomb branch while other parts remain better described in supergravity.

For charge vectors admitting bound states one can show from (3.8) that for every p there is at least one q such that $(\theta_p - \theta_q)(-1)^{s_{pq}} < 0$.¹³ For such pairs, the mass of the bifundamentals ϕ_{pq}^a will vanish and then become negative when $|\vec{x}_p - \vec{x}_q|$ is sufficiently small. In view of the scaling relation in section 4.1, this means that as $g_s \rightarrow 0$ some of the bifundamental chiral multiplets will become massless and then condense, taking the theory into the Higgs branch. Near the condensation point these fields are light and cannot be integrated out as in the analysis of the Coulomb branch. Indeed, since we are dealing with a one-dimensional effective Lagrangian, the wavefunction of a state can have a spread that overlaps the classical Higgs and Coulomb branches.¹⁴ We will not attempt the full quantum mechanical treatment of the wavefunction here (see [13] for some details) and instead analyze the classical Higgs branch that arises when the ϕ_{pq}^a have large VEVs.

In the classical Higgs branch, the vector multiplet scalars are set to zero — they acquire a mass from the Higgs mechanism and can be integrated out. From (4.21), the D-term equation $(\partial L_{\text{eff}}^V / \partial D_p)|_{D_p=0} = 0$ for supersymmetry gives the condition

$$\sum_q \sum_{a=1}^{|\gamma_{pq}|} |\phi_{pq}^a|^2 (-1)^{s_{pq}} = -\theta_p, \quad \forall p. \tag{4.31}$$

The solutions to this equation define the Higgs branch vacuum manifold. For example, if the quiver only has two nodes the vacuum manifold is $CP^{|\gamma_{pq}|-1}$. In general we obtain some intersection of complex projective spaces. A simple ansatz for solving these equations is to take all the bifundamentals between nodes p and q to have the same VEV

$$\phi_{pq}^a \equiv \frac{1}{2 R_{pq}}, \quad \forall a. \tag{4.32}$$

Then (4.31) becomes

$$\sum_q \frac{\gamma_{pq}}{2 R_{pq}} = -\theta_p. \tag{4.33}$$

Remarkably, this precisely reproduces the vacuum equation in the Coulomb branch (4.27) and the constraint equation in supergravity (4.29), suggesting how the classical moduli space of solutions can flow between these phases as g_s changes.

Matching the Coulomb and Higgs branches: At face value the classical Coulomb and Higgs branches each contain data that is absent in the other. In the Higgs branch, the ϕ_{pq}^a can each have independent VEVs and the ansatz (4.32) seems to only explore a simple subspace of the moduli space which reproduces the Coulomb branch. In the Coulomb branch the bifundamentals have been integrated out and the only piece that remains from their data are the multiplicities $|\gamma_{pq}|$. On the other hand, in the Coulomb branch, any solution to the constraints (4.29) must additionally satisfy triangle inequalities for the VEVs $\rho_{pq} = |\vec{x}_p - \vec{x}_q|$ ($\rho_{pq} + \rho_{ql} \geq \rho_{pl}$). These additional consistency conditions on a solution

¹³One can readily argue that if for some p , $(\theta_p - \theta_q)(-1)^{s_{pq}} \geq 0, \forall q$ then there is no solution to the constraint equation (3.8) or (4.28) for finite r_{pq} .

¹⁴Such overlaps were discussed in various contexts in [13, 50–52].

arise because the \vec{x}_p transform as vectors of $SO(3)$, the four dimensional rotation group. It is important to understand how such triangle inequalities can arise in the Higgs branch, since the D-term equations do not imply them. The bosonic fields ϕ_{pq}^a whose VEVs define the Higgs vacuum manifold are invariant under $SO(3)$. However, the fermionic partners of ϕ , produced by the action of a supercharge on ϕ_{pq}^a , transform in a spinor of $SO(3)$. This suggests that there is a further consistency condition on the Higgs branch vacua involving these fermions and the bosonic VEVs, but we have not identified this condition here. In addition, there is an $SU(2)$ action, called the Lefschetz $SU(2)$, on the cohomology of any Kähler moduli space. The latter is determined completely by the defining equation of the variety (4.31). Relating the Lefschetz $SU(2)$ to the spatial rotation group in the Coulomb branch, it seems possible that the triangle inequalities appear as some kind of global integrability condition on the manifold specified by values of ϕ_{pq}^a solving the D-term equations.

If the vacua in the Higgs and Coulomb branches can flow into each other as we are advocating, a minimal requirement is that the *number* of vacuum states in each branch should be equal. In the Higgs branch we must count the ground states of supersymmetric quantum mechanics on the classical moduli space (4.31). These are well-known to be in correspondence with the Dolbeault cohomology classes of the moduli space. Thus the number of supersymmetric ground states in the Higgs branch equals the sum of Betti numbers of the moduli space. For quivers without closed loops (and without extra adjoint matter) there is a formula from Reineke that computes these [53]. The corresponding problem in the Coulomb branch involves quantizing the motion of charged particles in the presence of monopoles (mutually nonlocal charges), and counting the resulting Landau levels. This has been done in some cases by Denef [13] and exactly matches the count of states in the Higgs branch. Interestingly, identical particle-monopole problems have appeared in recent approaches to counting the states of black holes and in the relation of such counting problems to topological string theory [54–56].

Finally, in our analysis we have separately studied the classical Higgs and Coulomb branches. In order to explicitly see a flow between them, we should construct the wavefunction in our quantum mechanical system and observe how it flows with changes of the coupling. Again, we refer to [13] for a detailed analysis in instructive special cases.

Non-abelian generalization and including adjoint chiral fields: We have focused on the case where all the $N_p = 1$. For more general values of N_p , we need to take a look at the effect of including the non-abelian $SU(N_p)$ degrees of freedom. For each node in our quiver, we split the set of N_p independent D-term equations into a singlet equation corresponding to the D-term in the center-of-mass $U(1)$ and $N_p - 1$ extra equations coming from the remaining $SU(N_p - 1)$ D-terms. The singlet equation is the only one which includes the FI term (\mathcal{F}_p) and the adjoint scalars (X_p^i , $i = 1, 2, 3$) do not appear; solving the singlet equations will thus involve exactly the same exercise as before. The $SU(N_p)$ equations take the form, written using the generators t_p^α ($\alpha = 1 \dots N_p^2 - 1$) [57]:

$$\forall \alpha : \quad \sum_q \sum_{a=1}^{|\gamma_{pq}|} Tr \left(t_p^\alpha \phi_{pq}^a \phi_{pq}^{a\dagger} \right) (-1)^{s_{pq}} = - \sum_{i=1}^3 Tr \left(t_p^\alpha [X_p^i, \bar{X}_p^i]^2 \right) . \quad (4.34)$$

These additional equations reflect the fact that a collection of N_p of our “atoms” is only classically situated at the single center which our singlet D-term equations see; quantum mechanically the N_p identical branes form a cloud of particles whose features are controlled by the same matrix Lagrangian that describes the interactions of N_p D0-branes [58, 59]. One of the key features of the D0-brane matrix Lagrangian is the condition that

$$[X^i, X^j] = 0 \quad \forall i \neq j, \tag{4.35}$$

which comes from minimizing the potential for the adjoint scalar fields. Since the adjoint chiral fields couple to the chiral bifundamentals through (4.34) we expect the bifundamentals to affect the internal dynamics of each “cloud” of N_p particles in some significant fashion; perhaps the perturbations in each cloud will be correlated via the singlet equations. This analysis is beyond the scope of our discussion, but we expect that the $SU(N_p)$ degrees of freedom will be important in giving the black hole its finite entropy and spatial size. This is an important difference in perspective compared to [13]. Finally, note that when $N_p > 1$, it is easy to see, at least in the two center case fully discussed in [13], that the overlap between the Higgs and Coulomb phases increases. This is reminiscent of the ideas in [50].

Superpotentials: The main limitation of the analysis presented above is that it does not include quivers with closed loops (e.g. figure 2). Such quivers are generic amongst black hole microstates, and will give rise to a superpotential for the chiral multiplets. While techniques for computing this superpotential for branes wrapped on a torus are available [60], the computation is done on a case by case basis, and will produce cubic and higher terms in the chiral multiplets. In the Coulomb branch these fields are massive, and the effective action is computed by integrating them out. Fortunately the contribution to the D-term equation from this computation is exact at one-loop and thus the superpotential plays no role in determining the Coulomb branch moduli space. Thus our description of a flow between supergravity and a gauge theory Coulomb branch as g_s is varied is unchanged. On the Higgs branch, however, a superpotential W will lead to a set of additional constraints, namely $\partial W / \partial \phi_{pq}^a = 0$, within the manifold defined by the D-term equation (4.31). While this will reduce the dimension of the classical Higgs moduli space, the number of quantum states could increase, decrease or remain unchanged depending on the cohomology of the constrained manifold. Unlike the case without closed loops [53], a general formula for the number of states in the Higgs branch is not available and hence the relation to the Coulomb branch in this case remains to be studied. In particular, while the analysis in [13] and above shows that states in the Coulomb branch will flow into the Higgs branch at very weak coupling, the converse is not obvious.

4.4 Summary and proposal

In [13], Denef suggests that states that flow from the Higgs branch into the Coulomb branch as the coupling is increased form a class of multi-center solutions separate from the black hole solutions that describe a large multiplicity of Higgs branch microstates. His reasons for suggesting this include: (a) the possibility of a complex Higgs branch topology leading to extra states, and (b) the fact that when there is a closed loop the Coulomb

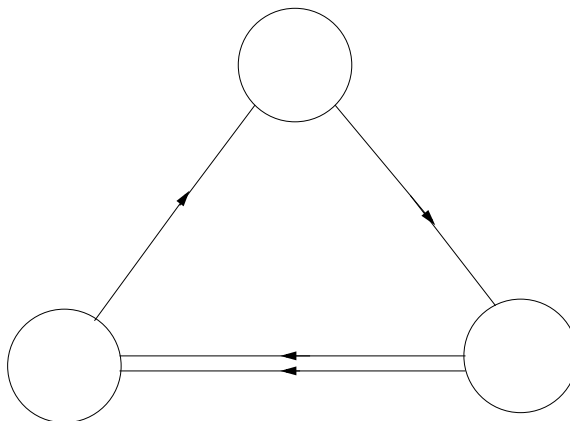


Figure 2: A quiver with a closed loop.

branch constraint equation (4.27) has a continuous family of solutions in which the centers approach each other ever more closely.¹⁵ The latter fact suggests that for any g_s there will be some states whose wavefunctions have substantial support on the Higgs branch. We are proposing a different perspective. In our view, the transverse growth of the size of a bound state as the system flows from Higgs branch to Coulomb branch to a closed string description is responsible for the formation of a complex macroscopic structure with an effective description as a black hole. In this perspective most microstates should enjoy such a flow and the usual solution with a horizon is simply the effective long-wavelength description of many complex, spatially extended microscopic bound states [3–6, 8–12].

5. Discussion

We make two proposals in this article. First, we suggest that every supersymmetric four dimensional black hole of finite area can be split up into microstates made of primitive 1/2-BPS “atoms”. The non-locality of the charges of these atoms binds these solutions together. Secondly, we propose that at very weak coupling these states appear as bound D-branes, but that as the coupling grows the bound state grows a transverse size leading to a topologically complex spacetime with an effective description as a black hole. At strong coupling the states form a “foam” in M-theory. To provide evidence for our proposal we constructed a large class of smooth, horizon-free supergravity solutions with the charges of four dimensional black holes, and demonstrated a scaling relation that takes them, as $g_s \rightarrow 0$, from a foam in M-theory, to multi-centered solutions in four-dimensional supergravity, to states in the D-brane gauge theory, first in the Coulomb branch and then in the Higgs branch. Our gauge theory analysis extensively used the results of Denef [13], who explicitly studies the flow of quantum mechanical wavefunctions from Coulomb to Higgs branch in some examples. We are also proposing that the reverse of this process, the flow of states from the Higgs branch into the Coulomb branch and then into a closed string description,

¹⁵An easy way to see this, is that the left hand side of all constraint equations will always involve at least two terms with opposite sign and so always have a solution for vanishing separation.

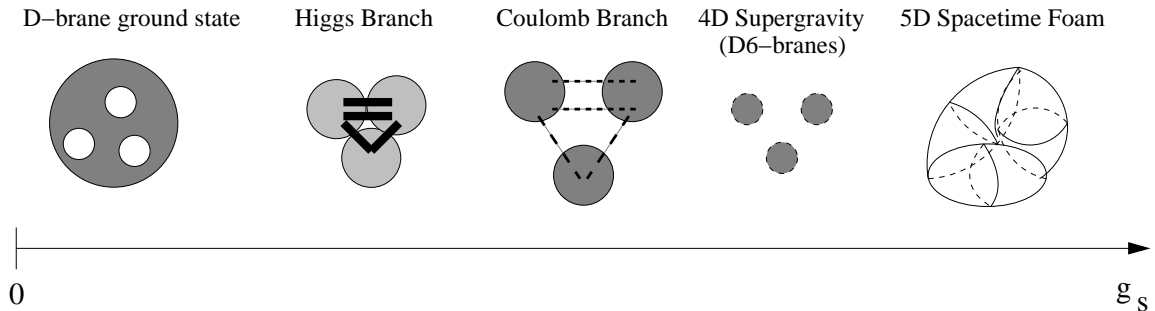


Figure 3: The different phases as we increase g_s .

is responsible for a transverse growth in the size of D-brane bound states as the string coupling increases, and that this is the link between the D-brane and “spacetime foam” pictures of black hole microstates.

To prove our proposals there are several further steps that must be taken

1. We must demonstrate that there are enough microstates constructed from 1/2-BPS “atoms” to account for the known entropy of the black hole carrying the total charge of the system.
2. We should show that the typical microstate at finite string coupling has a complex structure out to the horizon scale, but that the detailed microstructure is inaccessible to a conventional semiclassical observer.
3. We must complete our understanding of the relation of the Coulomb and Higgs branches of quiver gauge theories, in particular whether spacetime constraints such as triangle inequalities are realized in the Higgs branch also.
4. We must understand the role of the superpotentials that appear in quivers with closed loops in determining the structure of the Higgs branch moduli space, and whether and how this affects the flow of states between these branches as the coupling changes.

While these are challenging problems, solving them is important for understanding the quantum mechanical states underlying classical spacetimes.

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A. The central charge for $\alpha = 0$

The calculation of the $\mathcal{N} = 2$ central charge is outlined in [61] and is given by

$$Z_p = F_\mu p_p^\mu - X^\mu q_\mu^p, \tag{A.1}$$

where X^μ are the projective coordinates for the four-dimensional scalars

$$\begin{aligned} X^0 &= \frac{-1}{\sqrt{J_4}} \frac{\partial J_4}{\partial K^0} - iM_0 \rightarrow i \text{ as } \rho \rightarrow \infty, \\ X^i &= \frac{-1}{\sqrt{J_4}} \frac{\partial J_4}{\partial M_j} + iK^j \rightarrow (-1 + i\delta K^j). \end{aligned} \tag{A.2}$$

$F_\mu = \partial F(X)/\partial X^\mu$ is the derivative of the cubic prepotential¹⁶

$$F(X) = -\frac{X^1 X^2 X^3}{X^0}. \tag{A.3}$$

In the conventions of [61] the magnetic and electric charges for the p th brane are given by¹⁷

$$\begin{aligned} p^0 &= \frac{L}{2} n_p, & p^j &= \frac{1}{2} d_p^j \\ q_0 &= \frac{1}{2L^2} \frac{d_p^1 d_p^2 d_p^3}{n_p^2}, & q_i &= -\frac{1}{2L} s_{ijk} \frac{d_p^j d_p^k}{2n_p}. \end{aligned} \tag{A.4}$$

Putting this all together we find

$$Z_p = \frac{L}{2} n_p - \frac{1}{2L} \sum_{i,j,k} \frac{s_{ijk}}{2} n_p \lambda_p^j \lambda_p^k + \frac{i}{2} \Psi_p. \tag{A.5}$$

The mass of the p th brane is given by

$$m_p = \frac{|Z_p|}{4G_4}. \tag{A.6}$$

Note that the total central charge is $Z = \sum_p Z_p$, and that

$$Im[Z] = \frac{1}{2} \sum_p \Psi_p = 0 \tag{A.7}$$

because of the integrability constraint and the fact that we have set α in (3.3) to zero for simplicity.

¹⁶ F takes this simple form as a result of compactifying on T^6 and only turning on the eight charges of the STU model. A more general case will alter this expression in a straightforward manner.

¹⁷The reader will notice that our conventions exchange some electric and magnetic pairs by taking a series of Hodge duals.

B. Non-zero α

In general, for non-zero values of α the situation is more complicated as the reduction from five to four dimensions is now along a fiber of a slightly different magnitude (a similar situation arises in [23]).

Let us consider how $\alpha \neq 0$ affects the phase of the central charge Z as defined above. Note first that the expression $Im[Z_p] = \frac{1}{2}\Psi_p = 2\langle\Gamma_p, \Gamma_\infty\rangle$ is a trivial consequence of equations (A.1), (A.2) and the explicit form for the F_μ 's:

$$F_0 = \frac{+1}{\sqrt{J_4}} \frac{\partial J_4}{\partial M_0} + iK_0, \quad F_i = \frac{-1}{\sqrt{J_4}} \frac{\partial J_4}{\partial K^i} + iK_i. \quad (\text{B.1})$$

This implies that eq. (A.7) still holds, hence the central charge is real and α cannot be it's phase! It turns out that the phase α of the central charge used in [27, 29, 13] is defined in a different gauge, where a Kahler transformation has been used to fix the asymptotic value of $X^0 \rightarrow i$. As we will demonstrate, for non-zero α our asymptotic value for X_0 is $ie^{-i\alpha}$. Rotating this back to the Denef et al.'s gauge implies that the central charge picks up an overall phase of α . Hence α is the phase of the central charge in Denef et al.'s gauge. Note that the expression $\Psi_p = 4\langle\Gamma_p, \Gamma_\infty\rangle$ is gauge invariant. We will demonstrate that even for non-zero α the FI terms θ_p are stil exactly equal to $(8G_4)^{-1}\Psi_p$, as expected.

B.1 Redefining the harmonics for $\alpha \neq 0$

We start by observing that for non-zero α as defined in eq. (3.3) the asymptotic value of k_0 becomes $\frac{L}{2} \sin \alpha$. This implies that if we left our definitions for the harmonic functions M^μ, K_μ unchanged, the asymptotic value for J_4 would now be $\cos^2 \alpha < 1$. To remedy this situation we need to adjust the reduction of our five-dimensional solution to four dimensions as follows. First we recognize that the IIA coupling constant is now $g_s l_s = L \cos \alpha = R$, but we still define the new radial coordinate $\rho = 2r/L$. The time coordinate now also needs to be rescaled $t^{(4D)} = \sec \alpha t^{(5D)}$. The new harmonic functions are:

$$M_0 = -\cos \alpha HL^2/4, \quad K^0 = \sec^2 \alpha 4K/L, \quad K^i = Lh^i, \quad M_i = \sec \alpha M_i^{5D}, \quad (\text{B.2})$$

with similar scalings for Γ . With these definitions, $J_4(x) = I_4(\mathcal{H}(x)) \rightarrow 1$ at ∞ . The asymptotic value for X^0 is now $X^0 \rightarrow \sin \alpha + i \cos \alpha = ie^{-i\alpha}$ as previously advertised.

B.2 Checking the FI term

In the gauge of Denef et al., it also possible to write down the individual contribution to the central charge from each center as:

$$Z_p = \frac{R}{2} n_p \prod_i (1 + iB_p^i) = 4G_4 m_p e^{i\alpha_p}, \quad (\text{B.3})$$

where the $B_p^i = \left(\frac{d_p^i}{Rn_p} - \frac{\delta K^i}{\cos \alpha} + \frac{\sin \alpha}{\cos \alpha} \right)$ are the normalized (F-B) terms on each $D6$ -brane. This allows to quickly check that we have the right definition for the FI term:

$$\begin{aligned}
 (8G_4) \theta_p &= 2|Z_p| \sin(\alpha_p - \alpha) & (B.4) \\
 &= 2|Z_p| \sin \alpha_p \cos \alpha - 2|Z_p| \cos \alpha_p \sin \alpha \\
 &= Rn_p \operatorname{Re} \left[\prod_i (1 + iB_p^i) \right] \cos \alpha - Rn_p \operatorname{Im} \left[\prod_i (1 + iB_p^i) \right] \sin \alpha \\
 &= Rn_p \left[\sum_i B_p^i - \prod_i B_p^i \right] \cos \alpha - Rn_p \left[1 + \sum_i \frac{s_{ijk}}{2} B_p^j B_p^k \right] \sin \alpha \\
 &= \frac{n_p}{\cos \alpha} \left(\sum_i \lambda_p^i - \frac{1}{L^2} \prod_i \lambda_p^i + 2L \sin \alpha \right) \\
 &= \Psi_p^{4D} = 4 \langle \Gamma_p, \Gamma_\infty \rangle = \Psi_p^{5D} / \cos \alpha. & (B.5)
 \end{aligned}$$

Here we have differentiated $\Psi_p^{4D} = 4 \langle \Gamma_p, \Gamma_\infty \rangle$ which is defined canonically in $D = 4$ from the renormalized charge vectors Γ_p from 5D inspired Ψ_p^{5D} defined by simply adding a correction term ($2Ln_p \sin \alpha$) to the right-hand side of eq. (3.6).

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